# MEASUREMENTS OF THE PS LOW-FREQUENCY INDUCTIVE BROAD-BAND IMPEDANCE

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- **Introduction**
- **3 different methods used**
  - **Potential-well bunch lengthening (in 2000)**
  - **Coherent transverse tune shifts (in 2000 and 2001)**
  - **Incoherent quadrupole synchrotron frequency (in 2000)**
- **Other possible methods**
- **Other impedance models**
- **Conclusion**

# INTRODUCTION (1/2)

⇒ **Main components of the impedance of a standard circular machine**

- **Resistive-wall impedance**

- **Space-charge impedance**

- **Narrow-band resonators**

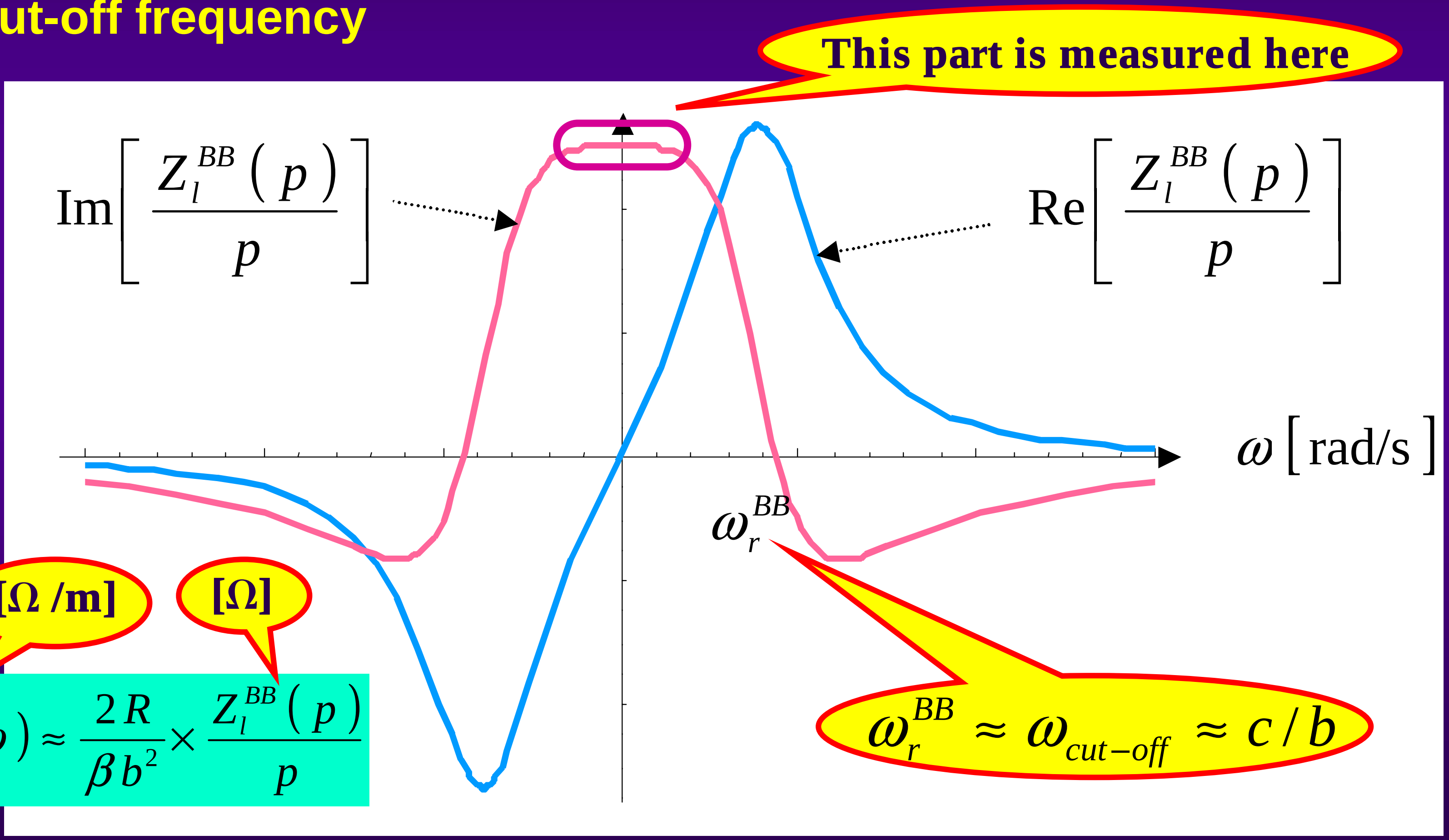
  ⇒ **parasitic high *Q* resonators due to higher order modes (HOMs) in the RF cavities for instance**

- **"Broad-band" impedance**

**- Behaviour discovered by J. Gareyte and F. Sacherer in 1974 through head-tail growth rate measurements in the CERN PS.**
**- Model (*Q* = 1 resonator ) first introduced by A. Hofmann and J.R. Maidment in 1979 in LEP**

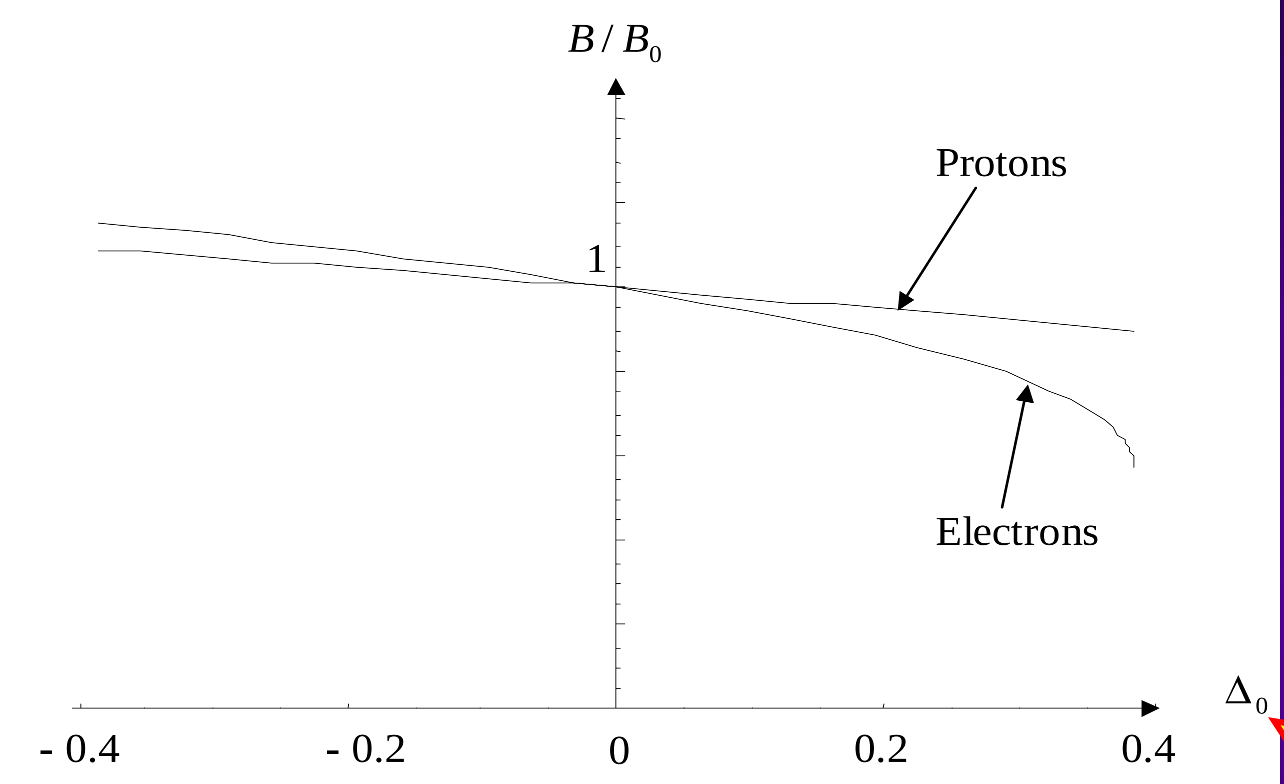
# INTRODUCTION (2/2)

- **The Broad-band impedance model : The averaged effect of the numerous changes in vacuum chamber cross-section (step changes, bellows, tanks, electrodes…) can be approximated by a low *Q* ~ 1 resonator with a resonance frequency equal to the pipe cut-off frequency**




$$Z_t^{BB}(\omega) \approx \frac{2R}{\beta b^2} \times \frac{Z_l^{BB}(p)}{p}$$

# POTENTIAL-WELL BUNCH LENGTHENING (1/3)




- **For** $\Delta_0 \leq 0$

$$\frac{B}{B_0} \approx 1 - \alpha_{P,E} \times \Delta_0$$

$$\begin{cases} \alpha_P = 0.25 \\ \alpha_E = 0.42 \end{cases}$$

$$\Delta_0 = \left( \frac{\hat{V}_T - \hat{V}_{RF}}{\hat{V}_{RF}} \right)_{B = B_0}$$

$$\mathrm{Im}\left[ \frac{Z_l(p)}{p} \right]_{SC+BB} = -\cos\phi_s \times \alpha'_{meas} \times \frac{\pi^2 \hat{V}_{RF}\, h \left( f_0 \, \tau_b^0 \right)^2}{3\, e} \times \begin{cases} 1 / \left( 2\alpha_E \right) & \text{for} \quad \text{electrons} \\ 1 / \alpha_P & \text{for} \quad \text{protons} \end{cases}$$

$$\alpha'_{meas} = d\tau_b / dN_b$$

## POTENTIAL-WELL BUNCH LENGTHENING (2/3)

- **The measurements have been performed with positrons at 3.5 GeV/c**

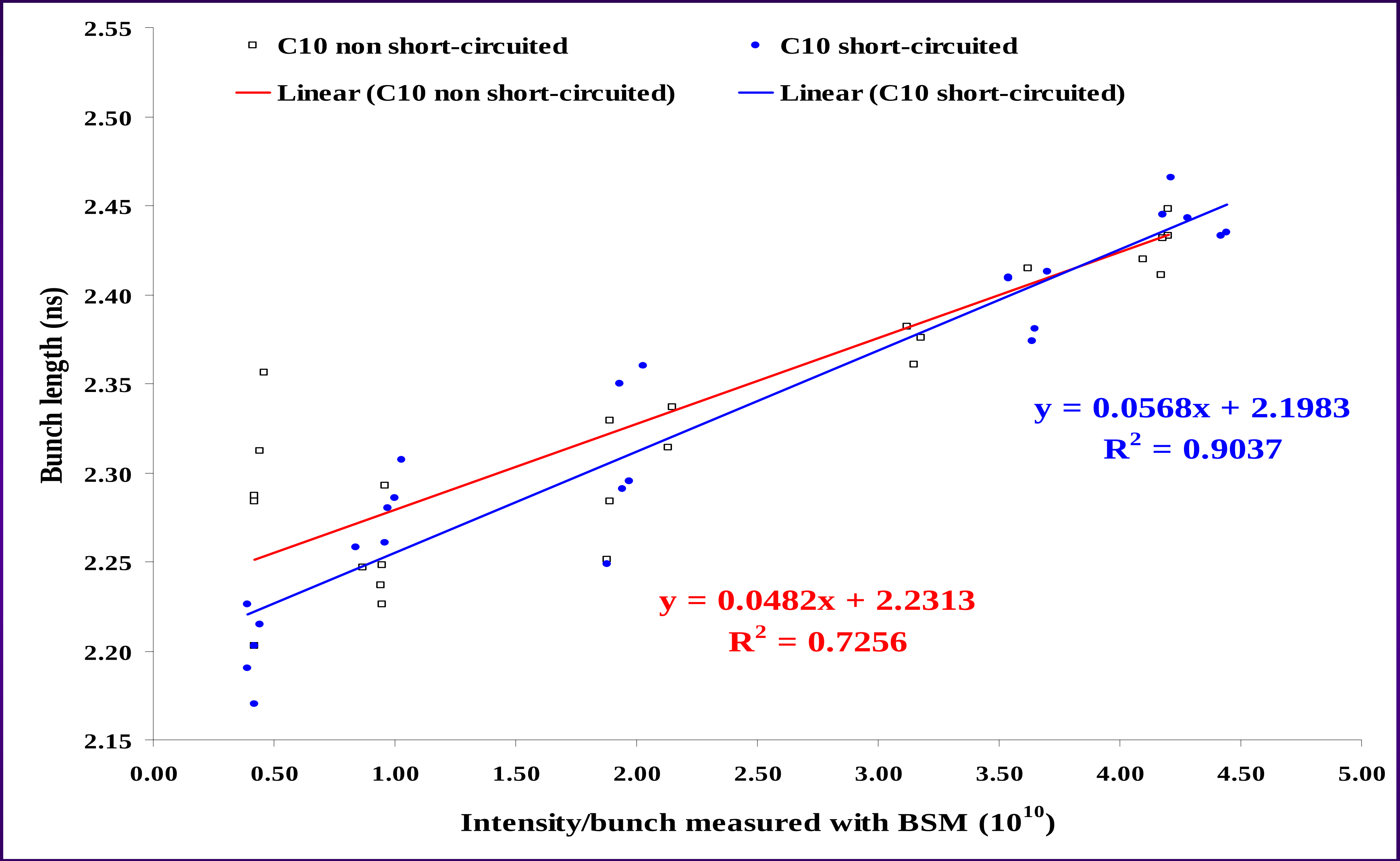
$\Rightarrow$ $\boxed{\cos\phi_s = -1}$

$\Rightarrow$ **No space charge**

$\boxed{\hat{V}_{RF} = 400 \text{ kV}}$ $\boxed{h = 240}$ $\boxed{f_0 = 477.135 \text{ kHz}}$

# POTENTIAL-WELL BUNCH LENGTHENING (3/3)

in 2000

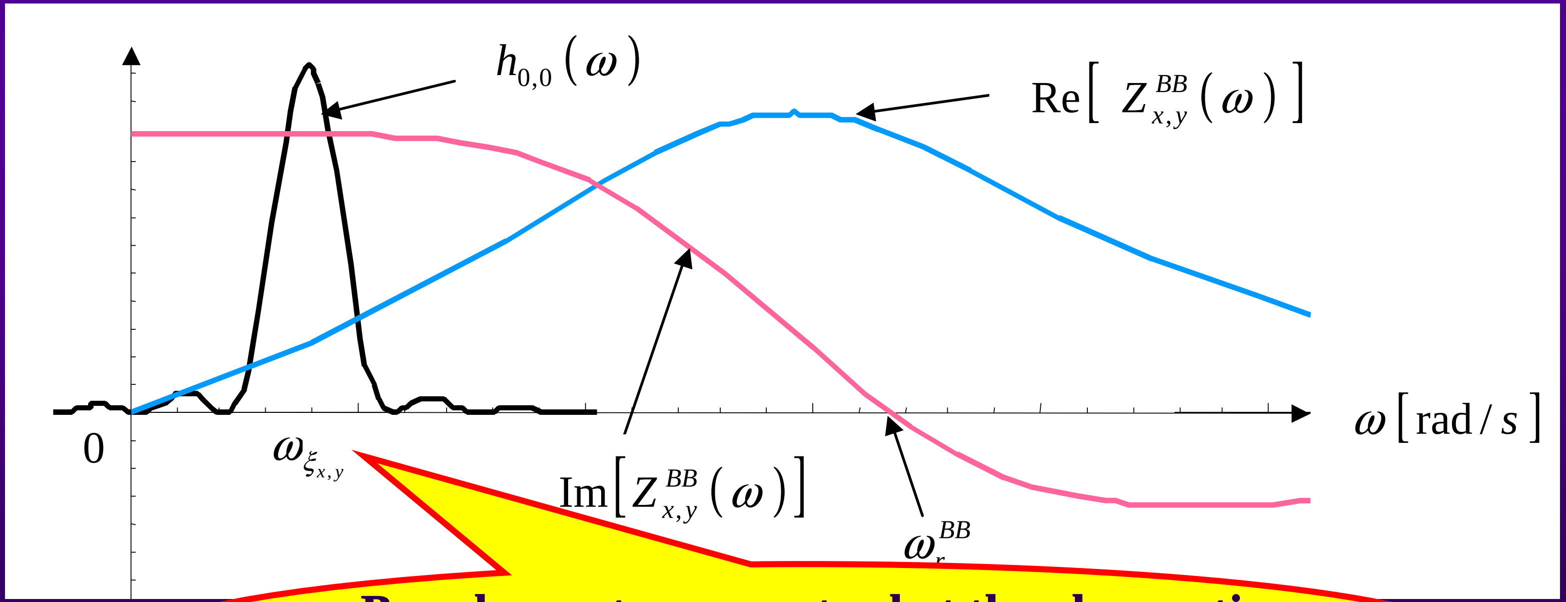



$$\Rightarrow \quad \mathrm{Im}\left[ Z_l^{BB}(p)/p \right] \approx 14 \pm 1\ \Omega$$

# COHERENT TRANSVERSE TUNE SHIFTS (1/5)

- **Sacherer formula for mode *m* = 0**

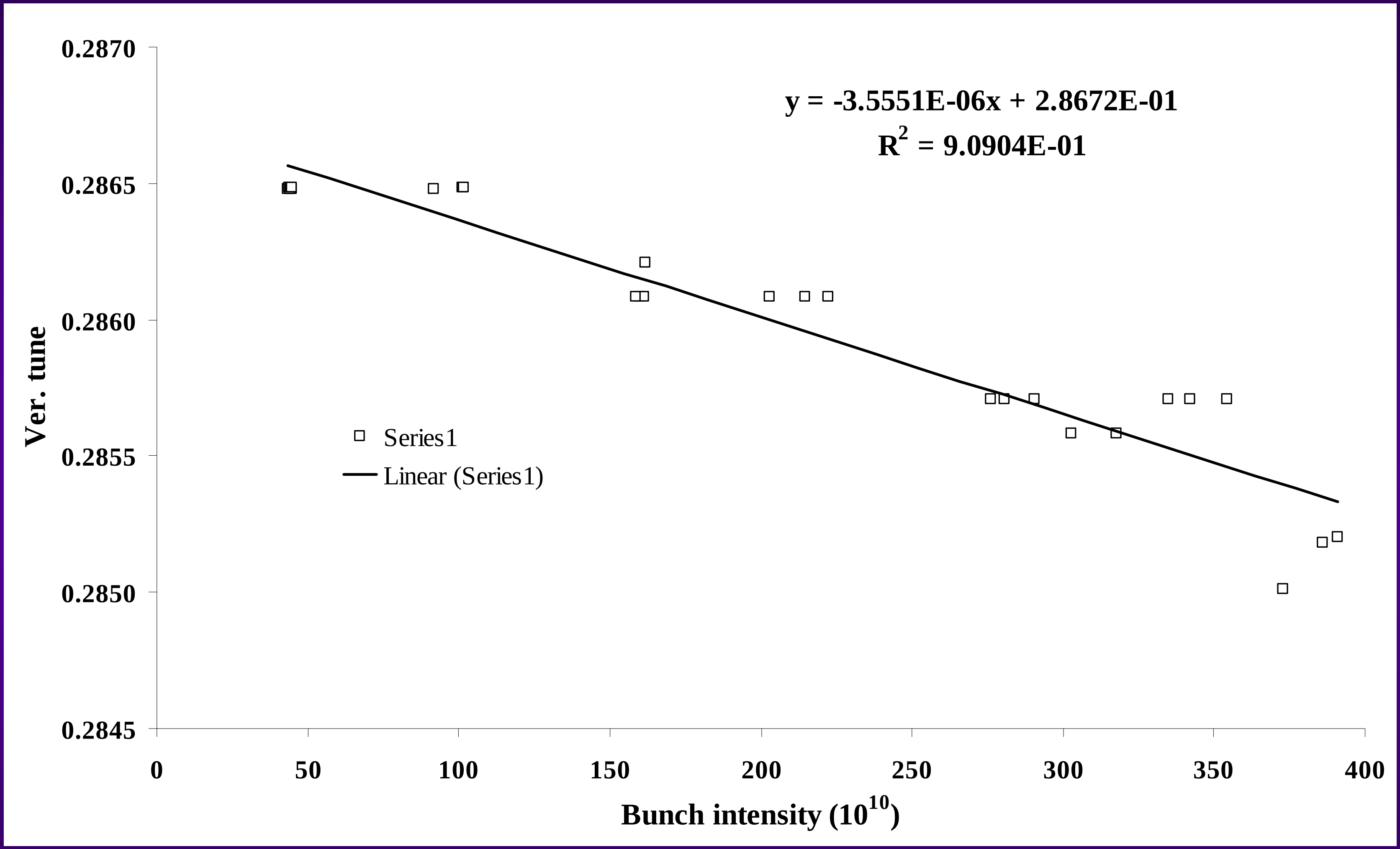
$$\Delta Q_{x,y}^{coh} = - \frac{e\,\beta\, I_b}{2\, m_0\, \gamma\, Q_{x0,y0}\, \Omega_0^2\, L} \times \mathrm{Im}\left[ \frac{\sum\limits_{k=-\infty}^{k=+\infty} Z_{x,y}^{BB}\left(\omega_k^{x,y}\right) h_{0,0}\left(\omega_k^{x,y} - \omega_{\xi_{x,y}}\right)}{\sum\limits_{k=-\infty}^{k=+\infty} h_{0,0}\left(\omega_k^{x,y} - \omega_{\xi_{x,y}}\right)} \right]$$




**Bunch spectrum, centred at the chromatic frequency, which extends to ~ ± 2$\pi$ /$\tau_b$**

# COHERENT TRANSVERSE TUNE SHIFTS (2/5)

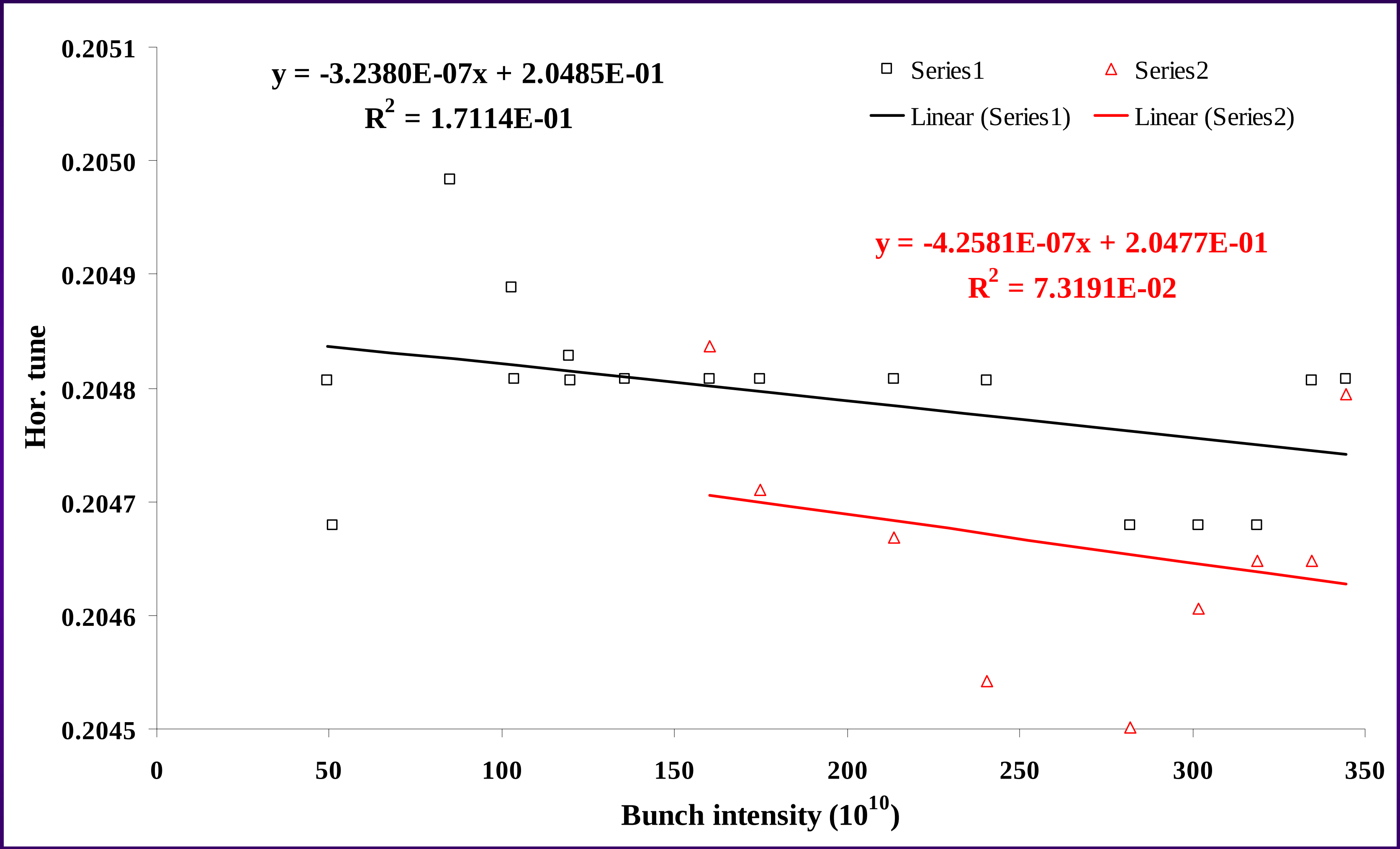
in 2000




$$\Rightarrow \quad \mathrm{Im}\left[ Z_l^{BB}(p) / p \right] \approx 20 \pm 3\ \Omega$$

# COHERENT TRANSVERSE TUNE SHIFTS (3/5)

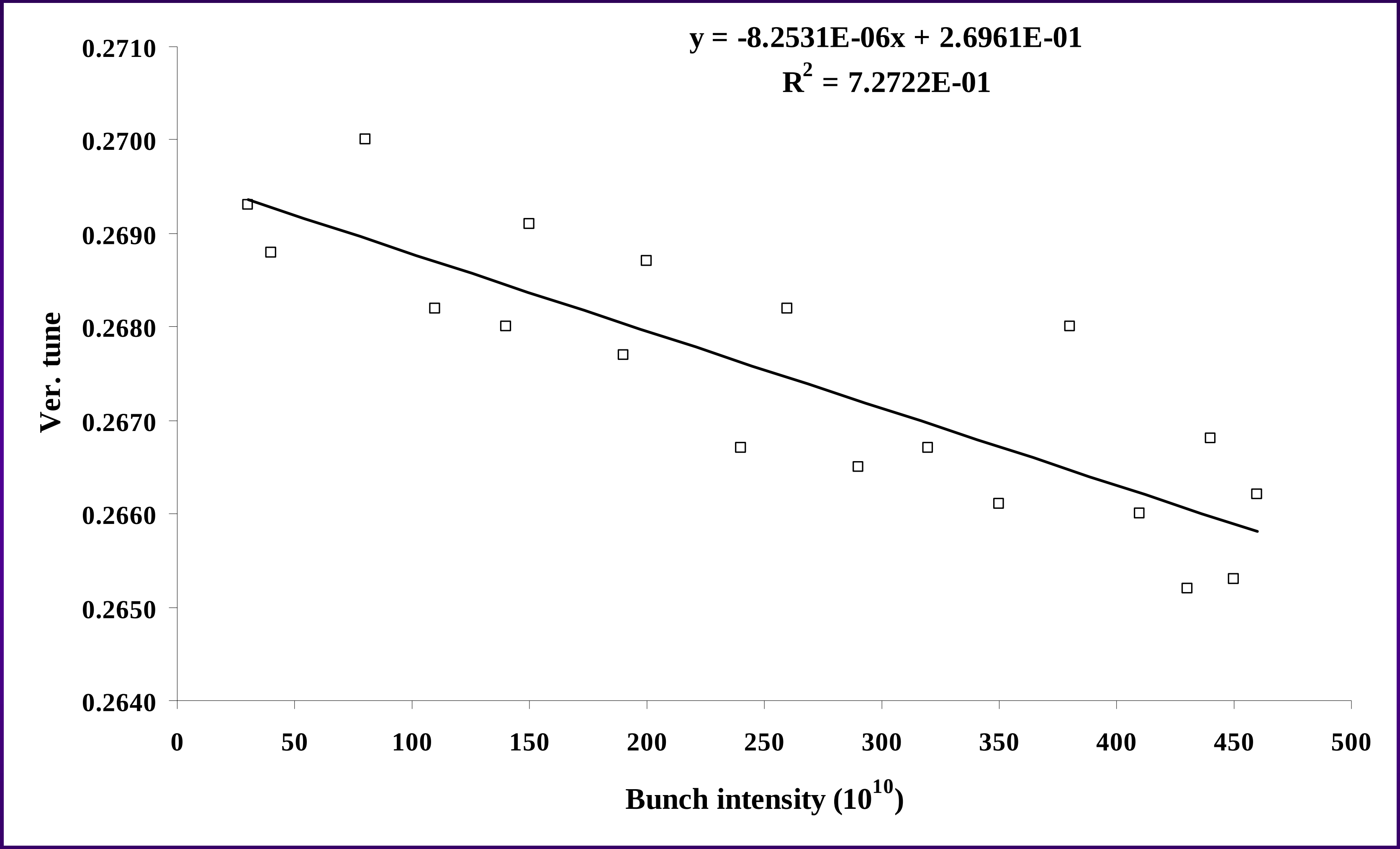
in 2000




⇒ **To be re-done…**

# COHERENT TRANSVERSE TUNE SHIFTS (4/5)

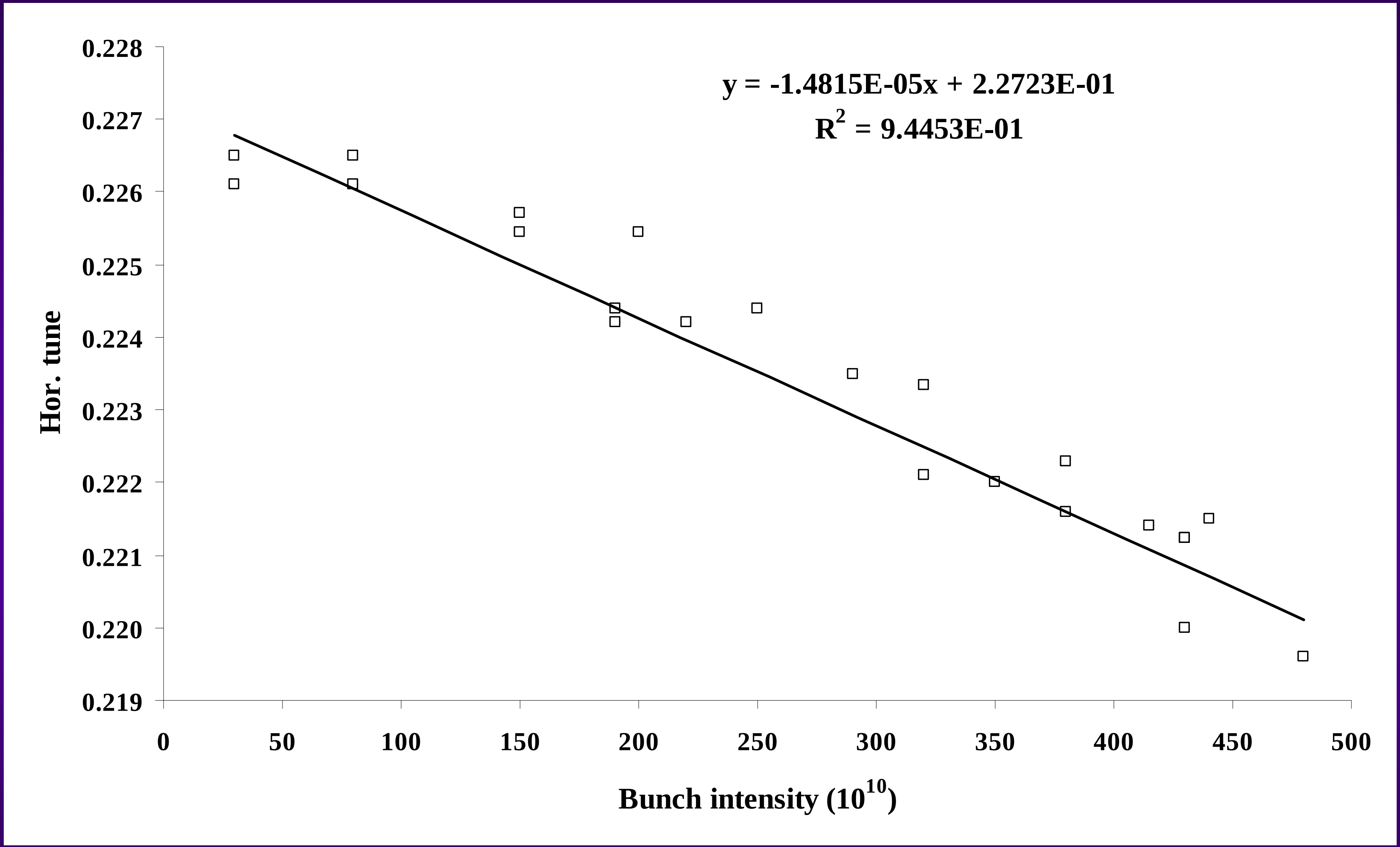
in 2001




$\Rightarrow$ $$\mathrm{Im}\left[ Z_l^{BB}(p) / p \right] \approx 46 \pm 16 \ \Omega$$

# COHERENT TRANSVERSE TUNE SHIFTS (5/5)

in 2001




$$\Rightarrow \boxed{\mathrm{Im}\left[ Z_l^{BB}(p) / p \right] \approx 186 \pm 14\ \Omega}$$ **???**

# INCOHERENT QUADRUPOLE SYNCHROTRON FREQUENCY (1/3)

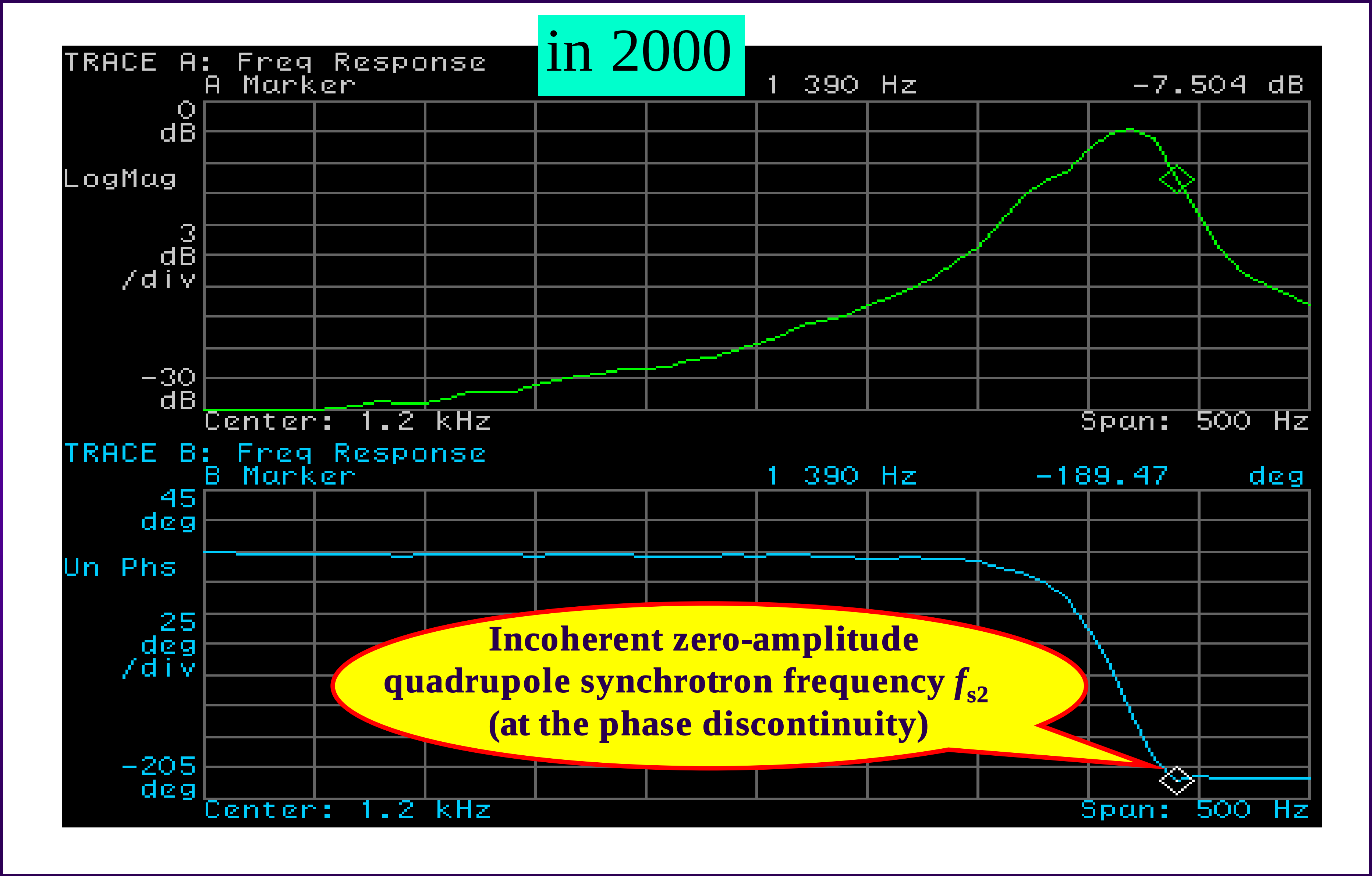
- **The beam was excited by band-limited noise (around the quadrupole synchrotron frequency) and its response obtained from the FFT of a time-domain measurement averaged over several shots**

- **The zero-amplitude frequency is at the upper limit of the phase response**

- **15 measurements in 3 distinct sets were included**

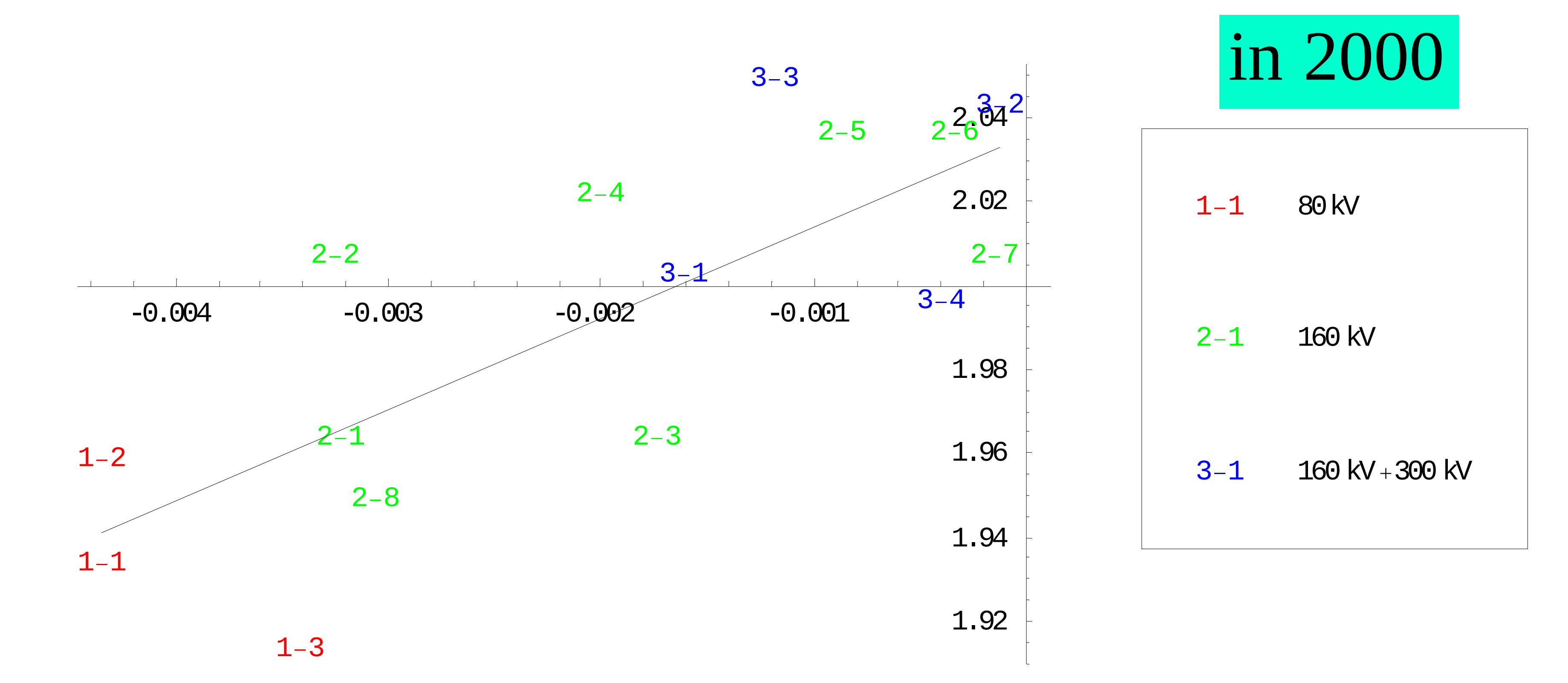
- **Over a range of different bunch lengths and intensities, 11 measurements were made in a single-harmonic (40 MHz) bucket and 4 in a dual-harmonic (40 + 80 MHz) bucket**

# INCOHERENT QUADRUPOLE SYNCHROTRON FREQUENCY (2/3)



# INCOHERENT QUADRUPOLE SYNCHROTRON FREQUENCY (3/3)

$$\frac{f_{s2}}{f_{s0}} = 2 + \frac{12\, e\, N_b}{\tau_b^3\, h\, \Omega_0^2 \left(V_1 \cos\phi_s - 2 V_2 \cos 2\phi_s\right)} \times \left| \frac{Z_l^{BB}(p)}{p} \right|$$




$\Rightarrow$ $\mathrm{Im}\left[ Z_l^{BB}(p) / p \right] \approx 21.7 \pm 5.1\ \Omega$

# OTHER POSSIBLE METHODS

- **Head-Tail growth rates**

**To measure** $\mathrm{Re}\left[ Z_l^{BB}(p) / p \right]$

- **Head-Tail decay rates (after a kick)**

- **Transverse Mode Coupling intensity threshold**

**Broad-band impedance ⇒ short-range wake-field**

- **Longitudinal microwave intensity threshold**

- **Parasitic loss of the beam = Energy lost when the beam traverses an impedance ⇒ Precise measurement of the longitudinal loss factor by measuring $Q_s$ vs. RF voltage for different intensities (LEP and SPS) ⇒ cf. Anke Susanne Muller**

# OTHER IMPEDANCE MODELS

- The broad-band resonator model addresses the impedance for frequencies $\omega \leq c / b$

- It is also possible to estimate the impedance at high frequencies $\omega >> c / b$ : the method used is called the diffraction model, and was first introduced by Lawson

$\Rightarrow$ Asymptotic frequency dependence of the broad-band impedance is $\omega^{-2}$ , while analytical study shows that the real part of the impedance decreases with frequency as $\omega^{-1/2}$ for a single cavity structure in an infinitely long smooth beam pipe, and as $\omega^{-3/2}$ for an array of periodic cavities $\Rightarrow$

  - Hofmann-Zotter impedance models
  - Heifets-Bane impedance model
  - Heifets-Chao impedance model
  - …

# CONCLUSION

- **Measurements in 2000 ⇒ Good agreement between the 3 methods**

$$\mathrm{Im}\left[ Z_l^{BB}(p)/p \right] \approx 20\,\Omega$$

**In agreement with the measurements done in the past**

**⇒ Bad measurement with the coherent horizontal tune shift**

- **Measurements in 2001 ⇒ Only 1 method used : Coherent transverse tune shifts, measured with the *Q*meter**

**⇒ The longitudinal impedance deduced from the vertical measurements is ~ 2 times bigger, and it is ~ 9 times bigger from the horizontal measurements**

**Did someone modify something ???**